Brain Activity Estimation using EEG-only Recordings Calibrated with joint EEG-fMRI Recordings using Compressive Sensing

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***Abstract*—Electroencephalogram (EEG) is a noninvasive, low-cost brain recording tool with high temporal but poor spatial resolution. In contrast, functional magnetic resonance imaging (fMRI) is a rather expensive brain recording tool with high spatial and poor temporal resolution. In this study, we aim at recovering the brain activity (source localization and activity-intensity) with high spatial resolution using only EEG recordings. Each EEG electrode records a linear combination of the activities of various parts of the brain. As a result, a multi-electrode EEG recording represents the brain activities via a linear mixing matrix. Due to distance attenuation, this matrix is almost sparse. Using simultaneous recordings of fMRI and EEG, we estimate the mixing matrix (calibration). Since Blood Oxygen Level Dependent (BOLD) signal of fMRI is a measure of energy used by active brain region, it has a quadratic relation with the electric potential waveform emitted from each fMRI *volume pixel* (voxel). Assuming uncorrelated time series from different regions, we reformulate the (underdetermined) forward problem as a linear problem and solve it using the Orthogonal Matching Pursuit (OMP) method. Besides the mixing matrix, the brain activities are often sparse spatially. Thus, we employ the estimated mixing matrix to extract the activity intensity of various brain regions from EEG recordings using iterative shrinkage thresholding algorithm (ISTA). We verify the proposed method on synthetic data. In particular, we divide the gray matter of the brain into 300 regions and assume a 30%-sparse measurement matrix, as well as 5% of regions to be active simultaneously. Simulations results show 88% accuracy in localizing the sources and and 66% accuracy in activity intensity estimation.**

***Index Terms*—Electroencephalogram, Blood Oxygen Level Dependency, Voxels, Gray matter, Orthogonal Matching Pursuit, Iterative Shrinkage Thresholding Algorithm**

I. Introduction

On one hand electroncephalogram (EEG) has become a widespread brain recording tool due to its noninvasive nature, relatively low-cost implementation and good temporal resolution. But it suffers from poor spatial resolution. On the other hand, functional magnetic resonance imaging (fMRI) provides a high spatial resolution with the expense of poor temporal resolution. In addition, fMRI require high initial and per-test costs. The type of experiments are also restricted by the movement limitations inside the MRI scanner. Therefore, it is highly desirable to use EEG instead of fMRI, if the spatial resolution of EEG is improved. Several research works are devoted to EEG source localization [1], [2]. In source localization problems, the location of the sources are investigated regardless of their activity intensity. In this paper, the source localization is achieved in two steps. In the first step, we identify the forward model; i.e., we estimate the effective impact of each brain voxel on each EEG electrode. In the second step, by having the forward model, we cast the source localization task as an inverse problem with sparsity constraint.

The electromagnetic wave emitted from each region is fairly modeled by an electrical dipole [3]. The reason is that the neural activities (spikes) coincide with strong electrical charge separation within a volume. the additivity of electric potential, at each EEG electrode we measure a weighted sum of net dipole moments of regions. Hence, EEG recordings are linked with regional activities via a mixing (measurement) matrix. Practically, the number of electrodes is far less than the number of brain voxels, which makes the measurement matrix extremely fat. The weights in partial sums depend on the distance between the electrodes and the brain voxels, as well as the permitivity of the inter-cell materials. Since the potential of a dipole decays quadratically with the distance, many entries of the measurement matrix are close to zero (negligible), and thus, the matrix can be assumed sparse. Therefore, it is logical to employ the technique in compressed sensing to estimate the mixing matrix.

Our main goal is to estimate the activity of each brain region or at least the location of active regions. Since only a fraction of the brain voxels is active at each time instance [1], the underdetermined system of equation form an inverse problem with sparsity constraint. On one hand, the activity magnitude of each region is defined as the number of spikes elicited in that region, in a short time period. The blood oxygen level dependent (BOLD) response of fMRI is more likely to represent this definition of activity (after being deconvolved from the hemodynamic response), since each spike consumes a certain amount of energy and the BOLD response is proportional to the total energy used by a region [4]. On the other hand, the net dipole moment associated with each region is the superposition of numerous dipoles (spikes from various neurons) which are not necessarily aligned, but more often scattered randomly [5]. In this paper, we use simultaneous EEG-fMRI recordings to estimate the measurement matrix. This is in contrast to the previous works on source localization which oftentimes use a loose approximation of the measurement matrix based on a permitivity map of the brain extracted from high-resolution MRI scans. We also show that the energy of the potential waveform of each region at given time intervals is proportional to the number of spikes elicited in that period. As a consequence, a quadratic relationship holds between the fMRI and EEG signals. Assuming the waveforms of coactivated regions to be uncorrelated, we reformulate the forward and inverse problems and solve them using OMP and ISTA respectively.

This paper is organized as follows. In Section II, we describe the mathematical model for production of synthetic data and show the relation between the activity of a region and the energy of its elicited waveform. We investigate the mathematical modeling of the forward and inverse problems of EEG source localization in Section III. The simulation results are reported in section IV. Finally, Section V concludes this paper.

II. Synthetic Data for Simultaneous EEG-fMRI

The electric potential of a dipole $\overrightarrow{p}$ located at the origin, observed at point $\overrightarrow{r}$ is given by:

$$V_p(\overrightarrow{r}) = k\frac{\overrightarrow{p}.\overrightarrow{r}}{\|r\|^3} = \frac{p.\cos\theta}{\|r\|^2} \tag{1}$$

where $k = \frac{1}{4\pi\epsilon}$ is the Coulomb's constant and θ is the angle between $\overrightarrow{p}$ and $\overrightarrow{r}$. We assign an electric dipole to each spike with a unit dipole moment. We also partition the gray matter into n regions and consider time intervals of T seconds. Assume the activity (the number of spikes in a time interval) of region i at a given time interval is b_i and these spikes are scattered randomly over the time interval. Then, the potential of region i at point $\overrightarrow{r}$ and time t is given by:

$$V_i\left(\overrightarrow{r},t\right) = \sum_{j=1:b_i} k\frac{\cos\theta_j}{\|r_j\|^2}\delta(t-t_j) = \frac{k}{\|r\|^2}\sum_{j=1:b_i}\delta(t-t_j).\cos\theta_j \tag{2}$$

where $\delta(t-t_j)$ indicates the occurrence of a spike at time t_j. Also note that at a point $\overrightarrow{r}$ sufficiently far from the region i (e.g., on the skull), all the dipoles within region i are almost at the same distance from $\|r\|$. Based on (2), we may also define the net dipole moment of region i as:

$$p_i(t) = \sum_{j=1:b_i}\delta(t-t_j).\cos\theta_j \tag{3}$$

Assuming a sufficiently large number of dipoles in each region, and uniform scattering of the directions in the space, the energy of this waveform within a time interval is given by:

$$E_i(\overrightarrow{r}) = \int_T V_i^2\left(\overrightarrow{r},t\right)dt = \frac{k^2}{\|r\|^4}\sum_{j=1:b_i}\cos^2\theta_j \approx \frac{k^2 b_i}{\|r\|^4}E_\theta\left[\cos^2\theta\right] = \frac{k^2 b_i}{2\|r\|^4} \tag{4}$$

We produce synthetic pairs of EEG-fMRI data as follows. For each region, a random integer between 40 and 2000 with steps of 40, is chosen as its activity. The spikes of each region are scattered randomly in a time interval of length $T = 2.5s$. The spikes are then assigned a random weight $\cos\theta$, with θ uniformly distributed in $[0, 2\pi]$. The resultant signal is the waveform of the net dipole moment associated with the region.

III. The Forward and inverse Problems via Compressed Sensing

A. The Forward Problem

As discussed earlier, the potential of electrode i at time t is a weighted sum of net dipole moments of regions:

$$e_i(t) = \sum_{j=1:n} W_{ij}p_j(t) \tag{5}$$

where, W_{ij} is the coefficient that links the net dipole in region j, to the recordings of electrode i. In a more accurate disciple, we may substitute the weights with some integrals, due to the changes in medium and consequently in ϵ and k.

The energy of $e_i(t)$ over a time interval is given by:

$$\varepsilon_i = \int_T e_i^2(t) = \sum_{j=1:n} W_{ij}^2\int_T p_j^2(t) + \sum_{j,k;j\neq k} W_{ij}W_{ik}\int_T p_j(t).p_k(t) \tag{6}$$

In the R.H.S of (6), the first integral equals $\frac{b_j}{2}$ and the second one is assumed to be zero since the waveform of coactive regions are assumed uncorrelated. Therefore:

$$\varepsilon_i \approx \sum_{j=1:n} W_{ij}^2 b_j \tag{7}$$

The matrix form of (7) is given by:

$$\mathbf{e} = \mathbf{M}\mathbf{b} \tag{8}$$

where $\mathbf{e} = [\varepsilon_1, \ldots, \varepsilon_E]$, E represents the number of electrodes, and $\mathbf{b} = [b_1, \ldots, b_n]$. The measurement matrix $\mathbf{M}$ is such that $M_{ij} = W_{ij}^2$.

For a set of N simulation measurements, we will have the pairs $\left(\mathbf{e}^1, \mathbf{b}^1\right), \ldots \left(\mathbf{e}^{\mathrm{N}}, \mathbf{b}^{\mathrm{N}}\right)$, by which, we tend to estimate the sparse unknown matrix $\mathbf{M}$. The resulting compressed sensing problem can be written as:

$$\begin{bmatrix} \mathbf{e}^1 \\ \vdots \\ \vdots \\ \mathbf{e}^N \end{bmatrix} = \begin{bmatrix} b_1^1 I_E & b_2^1 I_E & \cdots & b_n^1 I_E \\ b_1^2 I_E & \ddots & & b_n^2 I_E \\ \vdots & & \ddots & \vdots \\ b_1^N I_E & b_2^N I_E & \cdots & b_n^N I_E \end{bmatrix} \begin{bmatrix} M^1 \\ M^2 \\ \vdots \\ M^n \end{bmatrix};$$

$$I_E = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{bmatrix}_{E\times E} \tag{9}$$

where $b_j^i = \mathbf{b}^i(j)$ and M^i is the i^{th} column of $\mathbf{M}$. Now, the unknown matrix is reshaped as a vector. We further use OMP method to estimate the vectorized $\mathbf{M}$.

B. The inverse Problem

The inverse problem is simply given by (8), assuming the availability of the measurement matrix $\mathbf{M}$. We implemented a modified version of ISTA to retrieve the region activities $\mathbf{b}$. The modification is such that in each ISTA iteration, only the non-negative elements are preserved and the others are set to zero.

Also note that, the success of retrieving the spatial activities (at n voxels) depends on the RIP of the estimated matrix $\mathbf{M}$. Consequently, we can estimate the maximum feasible n by iteratively increasing n, estimating $\mathbf{M}$, and conducting its coherence.

IV. Simulations

We produced synthetic data for the simultaneous EEG-fMRI based on the description in section II, for a $n = 300$ number of regions and a $E = 64$ number of EEG electrodes. 5% of regions were assumed to be active simultaneously. The elements of $\mathbf{W}$were chosen according to a Laplace distribution and 70% of them were randomly set to zero. We used an $N = 200$ number of EEG-fMRI pairs to estimate $\mathbf{M}$. Using the estimated measurement matrix $\mathbf{M}$, we tested estimating regions' activities in 50 trials. Table 1 shows the simulation results. Also, fig.1 compares the original activities and the retrieved ones for a single trial. We measured the accuracy in activity estimation, by the percentage of regions whose activity was estimated we an error smaller than 30%.

TABLE I
Accuracy for localization and activity estimation

Table	**Accuracy**
Localization	0.88
Activity Estimation	0.66

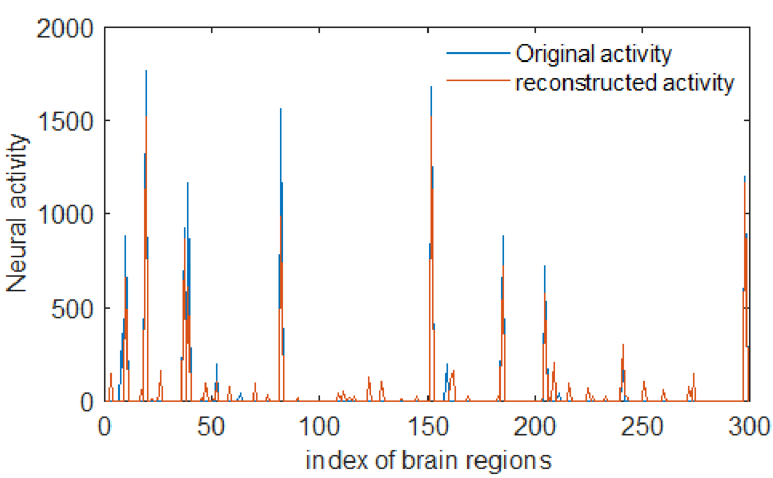

Fig. 1. Original and reconstructed neural activities

V. Conclusion

In this paper, we investigated solving the forward and inverse problems of EEG source localization, using compressed sensing approach. Using simultaneous EEG-fMRI recordings to solve the forward problem based on the compressed sensing, is a new idea and actually this paper has investigated the feasibility of the idea. While custom EEG source localization methods only estimate the locations of active sources, the proposed solution is capable of estimating the intensity of activity as well. Simulation results are promising and encourage testing the algorithm on real clinical data.

References

[1] E. Pirondini, B. Babadi , G. Obregon-Henao, C. Lamus, W. Q. Malik, M. S. Hamalainen, and P. L. Purdon "Computationally efficient algorithms for sparse, dynamic solutions to the EEG source localization problem," IEEE Transactions on Biomedical Engineering, vol. 65, no. 6, pp. 1359–1372, June 2018.

[2] B. He, L. Yang, C. Wilke, and H. Yuan, "Electrophysiological imaging of brain activity and connectivity-challenges and opportunities," IEEE Transactions on Biomedical Engineering, vol. 58, no. 7, pp. 1918 –1931, July 2011.

[3] Saeid Sanei, Adaptive processing of brain signals, Wiley, 2013.

[4] R. B. Buxton, "The physics of functional magnetic resonance imaging (fMRI)," Reports on Progress in Physics, vol. 76, no. 9, pp. 096601, September 2013.

[5] J. C. de Munck, P. C. M. Vijn, F. H. Lopes da Silva, "A random dipole model for spontaneous brain activity," IEEE Transactions on Biomedical Engineering, vol. 39, no. 8, pp. 791–804, August 1992.