Deep-Sparse Array Cognitive Radar

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***Abstract*—In antenna array based radar applications, it is often desirable to choose an optimum subarray from a full array to achieve a balance between hardware cost and resolution. Moreover, in a cognitive radar system, the sparse subarrays are chosen based on the target scenario at that instant. Recently, a deep-learning based antenna selection technique was proposed for a single target scenario. In this paper, we extend this approach to multiple targets and assess the performance of state-of-the-art direction of arrival estimation techniques in conjunction with the proposed antenna selection method. To optimally choose the subarrays based on the target DOAs, we design a convolutional neural network which accepts the array covariance matrix as an input and selects the best sparse subarray that minimizes the error. Once the optimum sparse subarray is obtained, the signals from the selected antennas are used to estimate the DOAs. We provide numerical simulations to validate the performance of the proposed cognitive array selection strategy. We show that the proposed approach outperforms random sparse antenna selection and it leads to a higher DOA estimation accuracy by 6 dB.**

I. Introduction

In a cognitive radar system, the resources such as bandwidth and number of antenna elements are often fixed. These resources are distributed among different radar systems based on the number of targets, noise levels, and clutter levels. Specifically, the resources are adaptively shared based on the current environment of the radars. For example, [1]–[3] developed cognitive radars where the available bandwidth is adaptively shared among different radars.

To accurately estimate the direction of arrival (DOA) of the moving targets, a large number of antennas are required to achieve high angular resolution. The radar structure with multiple antennas usually entails dedicated hardware equipment for each radar receive antennas which results in high cost. To reduce cost and power while obtaining sufficient performance, sparse array structures have been suggested where a subarray of the whole antenna array is utilized [4]–[7]. The use of subarrays allows cognitive operation, e.g., different subarrays can be selected to look into different directions simultaneously.

Reconfigurable array structures proposed in [4], [8], [9] use an adaptive switching matrix based on a combinatorial search for an optimal subarray that minimizes a lower bound on the DOA estimation error. In [10] and [11], the sparse array selection problem is cast as a convex optimization problem and an optimal antenna subarray is obtained for DOA estimation. Similarly, [12] selects the sensors in a distributed, multiple radar scenario through a greedy search with the Cramér-Rao lower bound (CRB) as a performance metric.

Nearly all of these formulations solve a mathematical optimization problem or use a greedy search algorithm. However, most of the antenna selection algorithms proposed in the literature have sub-optimum solutions [13]. Moreover, optimization-based and greedy-based methods have long computation times. To circumvent these issues, machine-learning based approaches are proposed in [7], [14]–[16]. In [14], a machine learning approach based on non-linear transformations of order statistics is applied for cognitive radar detection. A support vector machine (SVM) approach is applied to solve one-bit DOA estimation problems in [15] and antenna selection problems in communication scenarios in [16]. As a class of machine learning, deep learning (DL) has captured much interest recently to address many challenging problems such as speech recognition, visual object recognition, and language processing [17]. DL has several advantages such as low computational complexity while solving optimization-based or combinatorial search problems. It allows to extrapolate new features from a limited set of features contained in a training set [15], [17]. In the context of radar, DL is utilized in waveform recognition [18], image classification [19] and range-Doppler signature detection.

A DL solution for antenna selection was recently proposed in [7] for a single target scenario. In this paper, we extend the work in [7] to multiple targets. Specifically, we introduce a DL-based approach for sparse array selection in the context of passive sensing with multiple targets. To that end, we formulate the array selection problem for multiple targets as a classification task where each subarray designates a class. As in [7], we train a convolutional neural network (CNN) for finding the best subarray that leads to minimum mean-squared-error (MSE). The network input data is the covariance samples of the received array signal. To generate the training data for CNN, we compute the CRB for the given measurements and choose subarrays which lead to DOA estimation with the lowest minimal bound on the MSE. Hence we consider the minimization of the CRB as the performance benchmark in generating training sets. As we consider multiple targets, one can not directly use the CRB computed in [7]. Instead, in the training phase, we use the CRB for multiple sources as in [20].

In [7], the focus was on the antenna selection part. In this work, we compare the performance of two DOA estimation techniques, used with the proposed antenna selection method. To recover the DOAs, we exploit the signal sparsity and formulate a LASSO problem for recovering the signal power from the measurement covariance. We apply fast iterative soft-thresholding algorithm (FISTA) [21] to estimate DOAs and compare it with multiple signal classification algorithm (MUSIC) [22]. We then compare the performance of the proposed CNN-based subarrays with that of the full array and with random sub-arrays for different full array configurations. In particular for the choice of $D = 6$ out of an $N = 16$ element array, the full arrays perform 1 dB better than the subarrays due to a larger array aperture. Among the subarrays, the CNN-based subarray exhibits 6 dB better DOA estimation accuracy compared with a random subarray.

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II. Signal Model and Problem Formulation

A. Signal Model

Consider an N element phased array whose antennas are located at $\{\mathbf{v}_n\}_{n=1}^{N}$ where $\mathbf{v}_n = [x_n, y_n, z_n]^T$ indicates the antenna positions in a Cartesian coordinate system. Let K independent narrow-band sources with carrier wavelength λ impinge on the array from distinct DOAs $\{\Theta_k\}_{k=1}^{K}$ where $\Theta_k = (\theta_k, \phi_k)$ denotes the elevation and azimuth angle of the kth target respectively.

We denote the complex amplitude of the kth signal at snapshot time t as $x_k(t)$. Then the measurement vector $\mathbf{y}(t) = [y_1(t), y_2(t), ..., y_N(t)]^T$ received by the array can be expressed as

$$\mathbf{y}(t) = \sum_{k=1}^{K} \mathbf{a}(\Theta_k) x_k(t) + \mathbf{w}(t), \tag{1}$$

where $\mathbf{a}(\Theta_k)$ is a steering vector whose entries are given by

$$[\mathbf{a}(\Theta_k)]_n = \exp\left\{-j\frac{2\pi}{\lambda}\mathbf{v}_n^T \boldsymbol{\kappa}(\Theta_k)\right\}. \tag{2}$$

Here $\boldsymbol{\kappa}(\Theta_k) = [\sin(\phi_k)\cos(\theta_k), \cos(\phi_k)\cos(\theta_k), \cos(\theta_k)]^T$ denotes the target directions and the vector $\mathbf{w}(t) = [w_1(t), w_2(t), ..., w_N(t)]^T$ represents additive zero-mean white Gaussian noise.

We assume the spatial frequencies lie on a grid with M points where $K \ll M$. Thus, the received signal can be written as

$$\mathbf{y}(t) = \sum_{k=1}^{M} x_k(t)\mathbf{a}(\Theta_k) + \mathbf{w}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{w}(t), \tag{3}$$

where $\mathbf{A} = [\mathbf{a}(\Theta_1)\ \mathbf{a}(\Theta_2)\cdots\mathbf{a}(\Theta_K)]$ is the array manifold matrix and $\mathbf{x}(t) = [x_1(t), x_2(t), ..., x_M(t)]^T$ is a K-sparse vector, that is, it consists of only K nonzeros out of M. For the sake of clarity, hereafter we neglect the time index t and write

$$\mathbf{y} = \mathbf{A}\mathbf{x} + \mathbf{w}. \tag{4}$$

We further assume the sources $\mathbf{x}$ and the noise $\mathbf{w}$ to be zero-mean uncorrelated, i.e., $\mathbb{E}[\mathbf{x}\mathbf{x}^H] = \text{diag}(p_1, p_2, ..., p_M)$, $\mathbb{E}[\mathbf{w}\mathbf{w}^H] = p_w\mathbf{I}$ where $\mathbf{I}$ is the identity matrix, p_k and p_w are the powers of the kth source and of the noise, respectively. Under these assumptions, the covariance matrix of $\mathbf{y}$ can be written as

$$\mathbf{R}_y = \mathbb{E}[\mathbf{y}\mathbf{y}^H] = \sum_{k=1}^{M} p_k \mathbf{a}(\Theta_k)\mathbf{a}(\Theta_k)^H + p_w\mathbf{I}. \tag{5}$$

By vectorizing (5), we get the vector $\mathbf{r}$:

$$\mathbf{r} = \sum_{k=1}^{M} p_k \mathbf{b}(\Theta_k) + p_w \text{vec}(\mathbf{I}) = \mathbf{B}\mathbf{p} + p_w\text{vec}(\mathbf{I}), \tag{6}$$

where $\text{vec}(\mathbf{I})$ is the vectorization of $\mathbf{I}$ into a column stack. Here $\mathbf{B} = [\mathbf{b}(\Theta_1)\ \mathbf{b}(\Theta_2)\cdots\mathbf{b}(\Theta_M)]$ and $\mathbf{b}(\Theta) = \mathbf{a}(\Theta)\odot\mathbf{a}(\Theta)$ where $\odot$ denotes the Khatri-Rao product. The vector $\mathbf{p} = [p_1, p_2, ..., p_M]^T$ is a K-sparse vector with the same support as $\mathbf{x}$.

B. Problem Formulation

In this work, we aim to reduce the computational cost and energy of the radar system by utilizing fewer antennas. To that end, we consider a cognitive scan strategy where at the first scan all N antennas are active while at subsequent scans a subarray of $D < N$ antennas is used. To find the optimal subarray, we model the selection of D best antennas out of N as a classification problem wherein each class denotes an antenna subarray. Then, we train a CNN to select the best subarray based on the covariance of the signal received at the first scan [7]. Given the best subarray, we get upon reception an underdetermined linear system of equations. To solve the latter, we exploit the signal sparsity and formulate a LASSO problem for recovering the power signal. Once we obtain the power signal, we can determine the DOAs by exploiting the fact that the signal $\mathbf{x}$ and its power $\mathbf{p}$ share the same support.

III. Sparse Array Selection And Signal Processing

In this section, we introduce a method for sparse array selection and for DOA recovery given the chosen array. We first formulate the array selection as a classification problem and train a CNN to find the best sparse array based on the covariance matrix of the received signal following [7]. This allows to adaptively construct a sparse array. Given the sparse array obtained from the consecutive scans, we optimize the DOA recovery process.

A. Array Selection

Our aim is to select a D-element subarray from N-element full array. In this case, there are $Q = \binom{N}{D}$ possible choices. Hence this can be viewed as a classification problem with Q classes each of which represents a different subarray. Let $\mathcal{V}_d^q = \{v_{x_d}^q, v_{y_d}^q, v_{z_d}^q\}$ be the set of the dth antenna coordinates in Cartesian coordinate system for the qth subarray. Then, the qth class consisting of the positions of all elements in the qth subarray is $\mathcal{S}_q = \{\mathcal{V}_1^q, \ldots, \mathcal{V}_D^q\}$, and all classes are given by the set $\mathcal{S} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_Q\}$.

To label the training samples, we first compute the sample covariance matrix from L snapshots of noisy observations. Similar to [7], we use the CRB as a performance metric. However, due to multiple targets, the expression for the CRB is computed as in [20]. We denote the CRB as $\eta(\boldsymbol{\Theta}, \mathcal{S}_q)$ where $\boldsymbol{\Theta} \in \mathbb{R}^K$ denotes the target DOAs for all subarrays $q = 1, \ldots, Q$. The class labels for the input data indicate the best array, that is, the array which minimizes the CRB in a given scenario. Let us define $\bar{Q}$ as the number of subarrays that provide the best DOA estimation performance for different directions. Then, $\bar{Q}$ is generally much smaller than Q because of the direction of the targets and the aperture of the subarrays [7]. Hence, we construct a new set $\mathcal{L}$ which includes only those classes that represent the selected subarrays for different directions $\mathcal{L} = \{l_1, l_2, \ldots, l_{\bar{Q}}\}$, where $\bar{Q}$ is the reduced number of classes: $l_{\bar{q}}$ is the subarray class that provides the lowest CRB, namely

$$l_{\bar{q}} = \arg\min_{q=1,\ldots,Q} \eta(\boldsymbol{\Theta}, \mathcal{S}_q), \tag{7}$$

for $\bar{q} = 1, \ldots, \bar{Q}$. Once the label set $\mathcal{L}$ is obtained, the input-output data pairs are constructed as $(\mathbf{R}_y, z)$ where $z \in \mathcal{L}$ is the label representing the best subarray index for the covariance input $\mathbf{R}_y$. We summarize the steps for generating the training data in Algorithm 1. We use the sample covariance matrix as $\hat{\mathbf{R}}_y = \frac{1}{L}\sum_{l=1}^{L}\mathbf{y}(t)\mathbf{y}^H(t)$, which is generated with a signal-to-noise ratio $\text{SNR}_{\text{TRAIN}} = 10\log_{10}\left(\sigma_s^2/\sigma_n^2\right)$.

Using the labeled training dataset, we build a trained CNN classifier. The input of this learning system is the data covariance and the output is the index of the selected antenna set. Given the $N \times L$ output $\mathbf{Y}$ of the antenna array, the corresponding sample covariance is a complex-valued $N \times N$ matrix $\hat{\mathbf{R}}_y$. The features we consider in this work are the angle, real and imaginary parts of $\hat{\mathbf{R}}_y$. We construct three $N \times N$ real-valued matrices $\{\mathbf{X}_c\}_{c=1}^{3}$ whose (i, j)th entry contain, respectively, the phase, real and imaginary parts of the signal covariance matrix $\hat{\mathbf{R}}_y$: $[\mathbf{X}_1]_{i,j} = \angle[\hat{\mathbf{R}}_y]_{i,j}$; $[\mathbf{X}_2]_{i,j} = \mathbb{R}\text{e}\left\{[\hat{\mathbf{R}}_y]_{i,j}\right\}$; and $[\mathbf{X}_3]_{i,j} = \mathbb{I}\text{m}\left\{[\hat{\mathbf{R}}_y]_{i,j}\right\}$.

Algorithm 1 Training data generation for sparse array selection.

Input: Number of targets K, number of antennas N, sparse array size D, number of data realizations L, number of DOA angles P, number of snapshots T and $\text{SNR}_{\text{TRAIN}}$.

Output: Training data: Input-output pairs consisting of sample covariances $\hat{\mathbf{R}}_y^{(i,p)}$ and output labels $z_p^{(i)}$ for $p = 1, \ldots, P$ and $i = 1, \ldots, T$.

1: Select K target DOAs $\{\Theta_k^{(p)}\}_{k=1}^{K}$ for $p = 1, \ldots, P$.

2: Generate T different realizations of the array output $\{\mathbf{Y}_p^{(i)}\}_{i=1}^{T}$ for $p = 1, \ldots, P$ as $\mathbf{Y}_p^{(i)} = [\mathbf{y}_p^{(i)}(1), \ldots, \mathbf{y}_p^{(i)}(L)]$, where $\mathbf{y}_p^{(i)}(n) = \sum_{k=1}^{K} \mathbf{a}(\Theta_k^{(p)}) x_k^{(i)}(l) + \mathbf{w}^{(i)}(l)$, $x^{(i)}(l) \sim \mathcal{CN}(0, \sigma_s^2)$ and $\mathbf{w}^{(i)}(l) \sim \mathcal{CN}(0, \sigma_n^2 \mathbf{I})$.

3: Construct the input data $\hat{\mathbf{R}}_y^{(i,p)} = \mathbf{Y}_p^{(i)} \mathbf{Y}_p^{(i)^H} / L$.

4: Compute the CRB values $\eta(\boldsymbol{\Theta}, \mathcal{S}_q)$ and obtain the class set $\mathcal{L}$ representing the best subarrays.

5: Generate the input-output pairs as $(\hat{\mathbf{R}}_y^{(i,p)}, z_p^{(i)})$ for $p = 1, \ldots, P$ and $i = 1, \ldots, T$.

6: Construct training data by concatenating the input-output pairs:

$$\mathcal{D}_{\text{train}} = \{(\hat{\mathbf{R}}_y^{(1,1)}, z_1^{(1)}), (\hat{\mathbf{R}}_y^{(2,1)}, z_1^{(2)}), \ldots, (\hat{\mathbf{R}}_y^{(T,1)}, z_1^{(T)}), (\hat{\mathbf{R}}_y^{(1,2)}, z_2^{(1)}) \ldots, (\hat{\mathbf{R}}_y^{(T,P)}, z_P^{(T)})\},$$

where the size of the training dataset is $J = TP$.

We design a deep neural network composed of convolutional layers similar to that in [7]. The CNN consists of 9 layers. In the first layer, the CNN accepts the two-dimensional inputs $\{\mathbf{X}_c\}_{c=1}^{3}$ in three real-valued channels. The second, fourth and sixth layers are convolutional layers with 64 filters of size 2×2. The third and fifth layers are max-pooling to reduce the dimension by 2. The seventh and eighth layers are fully connected with 512 units whose 50% are randomly dropped out to reduce overfitting in training. There are rectified linear units (ReLU) after each convolutional and fully connected layers where $\text{ReLU}(x) = \max(x, 0)$. At the output layer, there are $\bar{Q}$ units wherein the network classifies the given input data using a softmax function and reports the probability distribution of the classes to provide the best subarray.

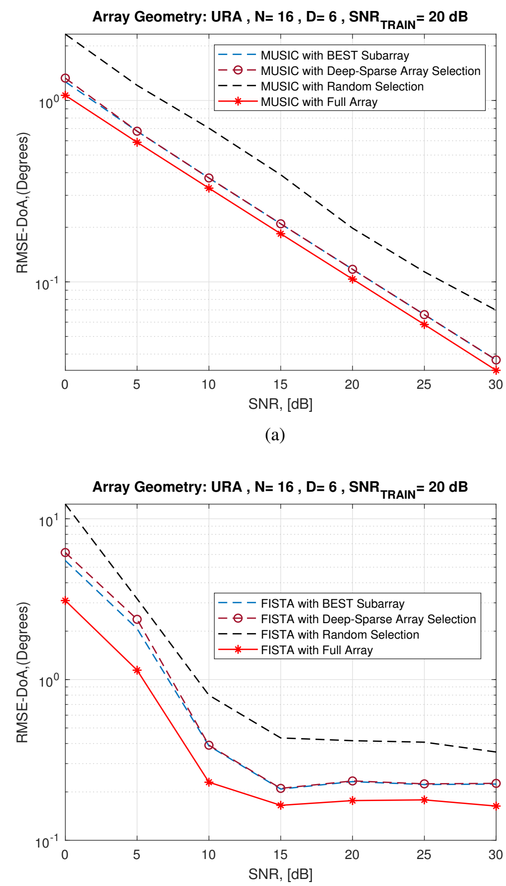
To train the network, we collect data for P target instances and for L realizations. Then, $\bar{Q}$, the number of subarrays providing the best DOA estimation performance is obtained. Note that $\bar{Q} \ll Q$ due to the structure of the array as reported in [7, Table 1]. Hence we obtain very few best subarray candidates even for large-scale antenna arrays where the complexity of (7) increases for large values of N and D. We realized the proposed network in MATLAB on a PC with 768-core GPU. During the training process, 90% and 10% of all data generated are selected as the training and validation datasets, respectively. We used the stochastic gradient descent algorithm with momentum for updating the network parameters with learning rate 0.05 and mini-batch size of 500 samples for 50 epochs.

B. DOA Estimation

Once the sparse array selection is performed, the chosen D antennas are utilized upon reception to yield the $D \times 1$ measurement signal $\bar{\mathbf{y}} = \bar{\mathbf{A}}\mathbf{x} + \bar{\mathbf{w}}$ where $\bar{\mathbf{A}} \in \mathbb{C}^{D \times M}$ is a sub-matrix of $\mathbf{A}$ in (4) whose rows are chosen according to the selected antennas. Following the same steps as in Section II, we compute the sample covariance matrix $\bar{\mathbf{R}}_y$ and by vectorizing it we get

$$\bar{\mathbf{r}} = \bar{\mathbf{B}}\mathbf{p} + p_w \text{vec}(\mathbf{I_D}) \tag{8}$$

where $\bar{\mathbf{B}} = \bar{\mathbf{A}} \odot \bar{\mathbf{A}}$ and $\mathbf{I_D}$ is a $D \times D$ identity matrix.



(a) (b)

Fig. 1. Performance comparison of different antenna selection techniques with a URA geometry: (a) DOA estimation by applying MUSIC and (b) DOA estimation by applying FISTA; CNN-based antenna selection method gives 6 dB less error compared with random antenna selection. For a given antenna selection method, estimation accuracy of MUSIC is 5 dB better than that of FISTA.

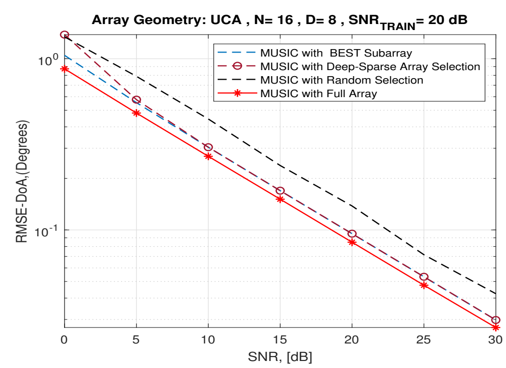


Fig. 2. Performance comparison of different antenna selection techniques with a UCA geometry with MUSIC technique based DOA estimation; CNN-based antenna selection method gives 6 dB less error compared with random antenna selection.

Next, we perform DOA estimation by using both MUSIC [22] and FISTA [21]. Assuming the number of sources K is known, the target DOAs can be found from the following MUSIC spectra

$$P(\Theta) = \frac{1}{\bar{\mathbf{a}}^H(\Theta)\mathbf{G}\mathbf{G}^H\bar{\mathbf{a}}(\Theta)}, \tag{9}$$

where $\bar{\mathbf{a}}(\Theta) \in \mathbb{C}^D$ is the steering vector and $\mathbf{G} \in \mathbb{C}^{D \times D-K}$ is the noise subspace eigenvector matrix of $\bar{\mathbf{R}}_y$. Then the DOA estimates can be obtained from K highest peaks of $P(\Theta)$. From (8), DOAs can also be estimated as a solution to the LASSO problem:

$$\min_{\mathbf{p}\geq\mathbf{0}} \quad ||\bar{\mathbf{r}} - \bar{\mathbf{B}}\mathbf{p}||_2^2 + \lambda||\mathbf{p}||_1, \tag{10}$$

where $\lambda \geq 0$ is chosen empirically. To solve (10) we use FISTA which iteratively updates the solution $\hat{\mathbf{p}}$. Once we recover the power signal, we estimate the DOAs by relying on the property that the signal and its power share the same support.

IV. Numerical Simulations

In this section, we evaluate the performance of our proposed DL approach for both sparse array selection and DOA estimation. We train the proposed CNN structure for $K = 2$ targets whose DOAs are uniform randomly selected in the interval $[0^\circ, 360^\circ]$. We select $P = 5000$ different target distributions and $L = 100$ data realizations with $T = 100$ data snapshots. The input data is prepared as described in Algorithm 1 and input-output data pairs are obtained. In order to test the trained network, a dataset, different from the training data, is generated with randomly selected target locations. When generating the target DOAs, we assume that the target DOAs are on the grid used for the DOA estimation algorithms as given in (3).

We consider both uniform rectangular array (URA) and uniform circular array (UCA) in our simulations with $N = 16$ antennas and half-wavelength array spacing. We select $T = 100$ data snapshots and the training SNR is selected as $\mathrm{SNR}_{\mathrm{TRAIN}} = 20$ dB. The performance of our CNN approach is compared with random array selection (RAS), full array and the best subarray, which is the subarray that provides the lowest MSE by the CRB computation in (7).

We assess the performance of the sparse array selection algorithms with both the MUSIC algorithm and the FISTA method. In Fig. 1, we present DOA estimation results for different algorithms for $D = 6$ where the reduced number of classes is calculated as $\bar{Q} = 29$ whereas $Q = 8008$. The MUSIC performance is shown in Fig. 1a and the FISTA performance is given in Fig. 1b. As seen, the proposed CNN approach effectively selects the best subarray for a large range of SNRs and it provides effective performance compared to RAS. For a given antenna selection method, comparing the DOA estimation performance of MUSIC and FISTA, it is observed that the MUSIC algorithm performs better than FISTA. The RMSE of FISTA does not reduce for high SNR while MUSIC provides much better precision. Specifically, the RMSE by MUSIC is 5 dB lower than that by FISTA.

We also provide the results for UCA geometry in Fig. 2 where the MUSIC algorithm is used for DOA estimation. We select $D = 8$ and $\bar{Q} = 32$ is obtained for this scenario. We obtain similar performance for UCA as in the URA results in Fig. 1. The CNN-based method has 6 dB lower RMSE compared with random antenna selection for different SNRs.

V. Conclusion

We proposed a deep-learning approach for sparse array selection for multiple targets. To that end, we cast the selection problem as a classification task and train a CNN to address it. We use the sample covariance matrix of the received signal, thus, allowing to perform cognitive selection of the sparse array. According to the network's output, we choose a sparse array which is used upon reception. Given the partial measurements received by the sparse array, we recover the DOAs by applying either MUSIC or FISTA. The specific choice of the solver related to whether the number of targets is known or unknown. The proposed approach can address multiple targets scenario and select the best array which leads to a low RMSE, outperforming random antenna selection.

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