ROBUST RECOVERY OF SPARSE NON-NEGATIVE WEIGHTS FROM MIXTURES OF POSITIVE-SEMIDEFINITE MATRICES

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Abstract. We consider a model where a matrix is generated as an s-sparse linear combination of d given nxn positive-semidefinite matrices. Recovering the unknown d-dimensional and s-sparse weights from noisy observations is an important problem in various fields of signal processing and also a relevant pre-processing step in covariance estimation. We will present related recovery guarantees and focus on the case of non-negative weights. The problem is formulated as a convex program and can be solved without further tuning. Such robust, non-Bayesian and parameter-free approaches are important for applications where prior distributions and further model parameters are unknown.

We will discuss some applications in wireless communication like estimating (non-negative) pathloss coefficients and user activity using multiple antennas. Here, a small subset of s«d devices indicate activity by transmitting specific length-n sequences which superimpose at each receive antenna with individual and unknown instantaneous channel coefficients. Well-known results in compressed sensing show that when using for given s and d sufficiently random sequences of length $n = O(s\,\mathrm{polylog}(d))$ one can recover per antenna w.h.p. the channel coefficients and the activity pattern (the essential support). However, since in future even s will grow considerably the question is how to further gain from a massive number of receive antennas.

We will present some recent ideas and scaling laws in this context. In particular, using the analysis above for the rank-one case, for given n and d one can recover pathloss coefficients and activity of up to $s = n^2/\mathrm{polylog}(d/n^2)$ devices from the empirical covariance over sufficiently many receive antennas.