The Convolution Word is Tied to the Exponential Kernel Transforms. What is a Parallel Expression for the Other Transforms?

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Abstract: Convolution products are very useful in computations involving exponential kernel transforms, such as the Fourier and Laplace transforms. The feasibility of this method is due to the important property of $e^{\alpha x} \cdot e^{\beta x} = e^{(\alpha+\beta)x}$, that is behind the simple form of the convolution product. Convolution also has the visual property, as it means bending together of the functions. For non-exponential kernel transforms, such as the Hankel transform, the analytical expressions of the Inverse Transform of Two Transforms Product (ITTTP) is quite complicated for use in practical analytical computations. Such difficulty is, mainly, due to the absence of the previously mentioned exponential function property. This is illustrated with a variety of well-known integral transforms. Also, such difficulty is supported by testimonials of experts in the field, such as Ruell Churchill and Ian Sneddon. Therefore, there is a need to return to basics numerical integration, according to the definition of the ITTTP. This is our most recent experience , in cooperation with M. Kamada, in trying to compute the general transform hill functions $\psi_{R+1}(x)$ associated with the Bessel function kernel, that the speaker had introduced in 1983. They are defined as the R-times J_N kernel "convolution parallel" of the gate function.

What is, then, a possible proposed name for this "convolution parallel" operation? It is, of course, left for the mathematical analysis community. However, it is important to note that Guiseppe Volterra (1860-1940) refrained from using the word convolution, and, instead, he used "Composition"! It is acceptable. Moreover, we, since 1972, have used, for the "generalized convolution", its generalized translation θ instead of the minus sign in the convolution product, and a generalized convolution theorem. D. Haimo, in 1954, also used the generalized convolution, and, rather recently, a member of authors have used this form. However, now, we do not feel that it is an appropriate expression.