



Thursday July 11, 1.45-8.00 pm

Wegener	1.45-2.45	<i>Lyapunov's theorem and sampling of continuous frames</i> Marcin Bownik Chair: Urbashi Mitra
<b>Graph signal processing</b> (invited session) Chair: Karlheinz Gröchenig & Isaac Pesenson		
A9 Amphi 1	2:50-3:15	<i>Sampling and reconstruction of graph signals: An overview of recent graph signal processing results</i> António G. Marques
	3.15-3.40	<i>Iterative Chebyshev Polynomial Algorithm for Signal Denoising on Graphs</i> Cheng Cheng, Junzheng Jiang, Nazar Emirov & Qiyu Sun
	3.40-4.05	<i>A non-commutative viewpoint on graph signal processing</i> Mahya Ghandehari, Dominique Guillot & Kristopher Hollingsworth
	4.05-4.30	<i>Coffee Break</i>
	4.30-4.55	<i>Clustering on Dynamic Graphs based on Total Variation</i> Peter Berger, Thomas Dittrich & Gerald Matz
	4.55-5.20	<i>Enabling Prediction via Multi-Layer Graph Inference and Sampling</i> Stefania Sardellitti, Sergio Barbarossa & Paolo Di Lorenzo
	5.20-5.45	<i>Blue-Noise Sampling of Signals on Graphs</i> Alejandro Parada, Daniel Lau, Jhony Giraldo & Gonzalo Arce
	5.45-6.10	<i>Average sampling, average splines and Poincaré inequality on combinatorial graphs</i> Isaac Pesenson
<b>7.30-11.00: Conference dinner</b> Café du Port		



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		<b>Wavelets, Shearlets,...</b> Chair: Andi Kivinukk
A9 Amphi 2	2:50-3:15	<i>Higher-dimensional wavelets and the Douglas-Rachford algorithm</i> Jeffrey Hogan, David Franklin & Matthew Tam
	3.15-3.40	<i>Analytic and directional wavelet packets</i> Valery Zheludev
	3.40-4.05	<i>Optimization in the construction of nearly cardinal and nearly symmetric wavelets</i> Neil Dizon, Jeffrey Hogan & Joseph Lakey
	4.05-4.30	<i>Coffee Break</i>
	4.30-4.55	<i>Analysis of shearlet coorbit spaces in arbitrary dimensions using coarse geometry</i> René Koch & Hartmut Führ
	4.55-5.20	<i>Trace Result of Shearlet Coorbit Spaces on Lines</i> Qaiser Jahan, Stephan Dahlke & Gabriele Steidl
		<b>7.30-11.00: Conference dinner</b> Café du Port



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		<b>Super-Resolution</b> Chair: Sinan Gunturk
A9 Amphi 3	2:50-3:15	<i>The dual approach to non-negative super-resolution: impact on primal reconstruction accuracy</i> Bogdan Toader, Stéphane Chrétiens & Andrew Thompson
	3.15-3.40	<i>Conditioning of restricted Fourier matrices and super-resolution of MUSIC</i> Wenjing Liao & Weilin Li
	3.04-4.05	<i>Iterative Discretization of Optimization Problems Related to Superresolution</i> Axel Flinth & Pierre Armand Weiss
		4.05-4.30 <i>Coffee Break</i>
		<b>Sampling and Fourier analysis</b> Chair: Arash Amini
A9 Amphi 3	4.30-4.55	<i>On the Reconstruction of a Class of Signals Bandlimited to a Disc</i> Ahmed Zayed
	4.55-5.20	<i>The Solvability Complexity Index of Sampling-based Hilbert Transform Approximations</i> Volker Pohl & Holger Boche
	5.20-5.45	<i>The Convolution Word is Tied to the Exponential Kernel Transforms. What is a Parallel Expression for the Other Transforms?</i> Abdul Jerri
		<b>7.30-11.00: Conference dinner</b> Café du Port



## Lyapunov's theorem and sampling of continuous frames

Marcin Bownik (University of Oregon, USA)

**Abstract:** In this talk we describe several recent developments stimulated by the solution of the Kadison-Singer Problem by Marcus, Spielman, and Srivastava. This includes an extension of Lyapunov's theorem for discrete frames due to Akemann and Weaver and a similar extension for continuous frames by the speaker. We also discuss the discretization problem posed by Ali, Antoine, and Gazeau, which asks whether a continuous frame can be sampled to obtain a discrete frame. This problem was recently solved by Freeman and Speegle using the solution of the Kadison-Singer problem. Generalizations of these results for trace class operator valued measures are also discussed.



## Graph Signal Processing (invited session)

Chair: Karlheinz Gröchenig & Isaac Pesenson

### 2.50-3.15: Sampling and reconstruction of graph signals: An overview of recent graph signal processing results

*Antonio G. Marques*

**Abstract:** Networks are structures that encode relationships between pairs of elements of a set. The simplicity of this definition drives the application of graphs and networks to a wide variety of disciplines. While often networks have intrinsic value and are themselves the object of study, in other occasions the object of interest is a signal defined on top of the graph, i.e., data associated with the nodes of the network. This is the matter addressed in the field of graph signal processing (GSP), where the notions of, e.g., frequency, filtering, or stationarity have been extended to signals supported on graphs. The goal of this talk is to review recent results on reconstruction of graph signals from observations taken at a subset of nodes. Leveraging the notions such as the Graph Fourier Transform and graph filters, we begin by analyzing the reconstruction under the assumption that the signal of interest lies on a known subspace which depends on the supporting graph. We then move to blind setups and describe efficient algorithms to address the reconstruction. The last part of the talk reviews kernel-based and non-linear approaches, establishing relations with semi supervised learning and matrix completion approaches.

### 3.15-3.40: Iterative Chebyshev Polynomial Algorithm for Signal Denoising on Graphs

Cheng Cheng, Junzheng Jiang, Nazar Emirov & *Qiyu Sun*

**Abstract:** In this paper, we consider the inverse graph filtering process when the original filter is a polynomial of some graph shift on a simple connected graph. The Chebyshev polynomial approximation of high order has been widely used to approximate the inverse filter. In this paper, we propose an iterative Chebyshev polynomial approximation (ICPA) algorithm to implement the inverse filtering procedure, which is feasible to eliminate the restoration error even using Chebyshev polynomial approximation of lower order. We also provide a detailed convergence analysis for the ICPA algorithm and a distributed implementation of the ICPA algorithm on a spatially distributed network. Numerical results are included to demonstrate the satisfactory performance of the ICPA algorithm in graph signal denoising.



## Graph Signal Processing (invited session)

Chair: Karlheinz Gröchenig & Isaac Pesenson

### 3.40-4.05: A non-commutative viewpoint on graph signal processing

*Mahya Ghandehari, Dominique Guillot & Kristopher Hollingsworth*

**Abstract:** The recent field of graph signal processing aims to develop analysis and processing techniques, designed for data that is best represented on irregular domains such as graphs. To this end, important notions of classical signal processing, such as smoothness, band-limitedness, and sampling, should be extended to the case of graph signals. One of the most fundamental concepts in classical signal processing is the Fourier transform. Recently, graph Fourier transform was defined as a generalization of the Fourier transform on Abelian groups, and many of its properties were investigated. However, a graph is usually the manifestation of a non-commutative structure; this can be easily seen in the case of the Cayley graph of a non-Abelian group. In this article, we investigate a new approach to develop concepts of Fourier analysis for graphs. Our point of view is inspired by the theory of non-commutative harmonic analysis, and is founded upon representation theory of non-Abelian groups.

### 4.30-4.55: Clustering on Dynamic Graphs based on Total Variation

*Peter Berger, Thomas Dittrich & Gerald Matz*

**Abstract:** We consider the problem of multiclass clustering on dynamic graphs. At each time instant the proposed algorithm performs local updates of the clusters in regions of nodes whose cluster affiliation is uncertain and may change. These local cluster updates are carried out through semi-supervised multiclass total variation (TV) based clustering. The resulting optimization problem is shown to be directly connected to a minimum cut and thus very well suited to capture local changes in the cluster structure. We propose an ADMM based algorithm for solving the TV minimization problem. Its per iteration complexity scales linearly with the number of edges present in the local areas under change and linearly with the number of clusters. We demonstrate the usefulness of our approach by tracking several objects in a video with static background.



## Graph Signal Processing (invited session)

Chair: Karlheinz Gröchenig & Isaac Pesenson

### 4.55-5.20: Enabling Prediction via Multi-Layer Graph Inference and Sampling

*Stefania Sardellitti, Sergio Barbarossa & Paolo Di Lorenzo*

**Abstract:** In this work we propose a novel method to efficiently predict dynamic signals over both space and time, exploiting the theory of sampling and recovery of band-limited graph signals. The approach hinges on a multi-layer graph topology, where each layer refers to a spatial map of points where the signal is observed at a given time, whereas different layers pertain to different time instants. Then, a dynamic learning method is employed to infer space-time relationships among data in order to find a band-limited representation of the observed signal over the multi-layer graph. Such a parsimonious representation is then instrumental to use sampling theory over graphs to predict the value of the signal on a future layer, based on the observations over the past graphs. The method is then tested on a real data-set, which contains the outgoing cellular data traffic over the city of Milan. Numerical simulations illustrate how the proposed approach is very efficient in predicting the calls activity over a grid of nodes at a given daily hour, based on the observations of previous traffic activity over both space and time.

### 5.20-5.45: Blue-Noise Sampling of Signals on Graphs

*Alejandro Parada, Daniel Lau, Jhony Giraldo & Gonzalo Arce*

**Abstract:** In the area of graph signal processing, a graph is a set of nodes arbitrarily connected by weighted links; a graph signal is a set of scalar values associated with each node; and sampling is the problem of selecting an optimal subset of nodes from which any graph signal can be reconstructed. For small graphs, finding the optimal sampling subset can be determined by looking at the graph's Fourier transform; however in some cases the spectral decomposition used to calculate the Fourier transform is not available. As such, this paper proposes the use of a spatial dithering, on the graph, as a way to conveniently find a statistically good, if not ideal, sampling - establishing that the best sampling patterns are the ones that are dominated by high frequency spectral components, creating a power spectrum referred to as blue-noise. The theoretical connection between blue-noise sampling on graphs and previous results in graph signal processing is also established, explaining the advantages of the proposed approach. Restricting our analysis to undirected and connected graphs, numerical tests are performed in order to compare the effectiveness of blue-noise sampling against other approaches.



## Graph Signal Processing (invited session)

Chair: Karlheinz Gröchenig & Isaac Pesenson

### 5.45-6.10: Average sampling, average splines and Poincare inequality on combinatorial graphs

*Isaac Pesenson*

**Abstract:** In the setting of a weighted combinatorial finite or infinite countable graph  $G$  we introduce functional Paley-Wiener spaces  $PW_\omega(L)$ ,  $\omega > 0$ , defined in terms of the spectral resolution of the combinatorial Laplace operator  $L$  in the space  $L^2(G)$ . It is shown that functions in certain  $PW_\omega(L)$ ,  $\omega > 0$ , are uniquely defined by their averages over some families of "small" subgraphs which form a cover of  $G$ . Reconstruction methods for reconstruction of an  $f \in PW_\omega(L)$ , from appropriate set of its averages are introduced. One method is using language of Hilbert frames. Another one is using average variational interpolating splines which are constructed in the setting of combinatorial graphs.



## Wavelets, Shearlets,...

Chair: Andi Kivinnuk

### 2.50-3.15: Higher-dimensional wavelets and the Douglas-Rachford algorithm

*Jeffrey Hogan, David Franklin & Matthew Tam*

**Abstract:** We recast the problem of multiresolution-based wavelet construction in one and higher dimensions as a feasibility problem with constraints which enforce desirable properties such as compact support, smoothness and orthogonality of integer shifts. By employing the Douglas-Rachford algorithm to solve this feasibility problem, we generate one-dimensional and non-separable two-dimensional multiresolution scaling functions and wavelets.

### 3.15-3.40: Analytic and directional wavelet packets

*Valery Zheludev*

**Abstract:** The paper presents a versatile library of analytic and quas-analytic complex-valued wavelet packets (WPs) which originate from discrete splines of arbitrary orders. The real parts of these WPs are the regular spline-based orthonormal WPs. The imaginary parts are the so-called complementary orthonormal WPs, which, unlike the symmetric regular WPs, are antisymmetric. Tensor products of 1D quasi-analytic provide a diversity of 2D WPs oriented in multiple directions. For example, the set of fourth-level WPs comprise 256 different directions. The designed computational scheme enables extremely fast and easy design and implementation of the WP transforms.

### 3.40-4.05: Optimization in the construction of nearly cardinal and nearly symmetric wavelets

*Neil Dizon, Jeffrey Hogan & Joseph Lakey*

**Abstract:** We present two approaches to the construction of scaling functions and wavelets that generate nearly cardinal and nearly symmetric wavelets on the line. The first approach casts wavelet construction as an optimization problem by imposing constraints on the integer samples of the scaling function and its associated wavelet and with an objective function that minimizes deviation from cardinality or symmetry. The second method is an extension of the feasibility approach by Franklin, Hogan, and Tam to allow for symmetry by considering variables generated from uniform samples of the quadrature mirror filter, and is solved via the Douglas-Rachford algorithm.



## Wavelets, Shearlets,...

Chair: Andi Kivinukk

### 4.30-4.55: Analysis of shearlet coorbit spaces in arbitrary dimensions using coarse geometry

*René Koch & Hartmut Führ*

**Abstract:** In order to analyze anisotropic information of signals, the shearlet transform has been introduced as class of directionally selective wavelet transform. One way of describing the approximation-theoretic properties of such generalized wavelet systems relies on coorbit spaces, i.e., spaces defined in terms of sparsity properties with respect to the system. In higher dimensions, there are several distinct possibilities for the definition of shearlet systems, and their approximation-theoretic properties are currently not well-understood. In this note, we investigate shearlet systems in higher dimensions derived from two particular classes of shearlet groups, the standard shearlet group and the Toeplitz shearlet group. We want to show that different groups lead to different approximation theories. The analysis of the associated coorbit spaces relies on an alternative description as decomposition spaces that was recently established. For a shearlet group, this identification is based on a covering of the associated dual orbit induced by the shearlet group. The geometry of the sets in this covering is the determining factor for the associated decomposition space. We will see that the orbit can be equipped with a metric structure that encodes essential properties of this covering. The orbit map then allows to compare the geometric properties of coverings induced by different groups without the need to explicitly compute the respective coverings, which gets increasingly difficult for higher dimensions. This argument relies on a rigidity theorem which states that geometrically incompatible coverings lead to different decomposition spaces in almost all cases.



## Wavelets, Shearlets,...

Chair: Andi Kivinukk

### 4.55-5.20: Trace Result of Shearlet Coorbit Spaces on Lines

*Qaiser Jahan*, Stephan Dahlke & Gabriele Steidl

**Abstract:** We study traces of certain subspaces of shearlet coorbit spaces on lines in  $R^2$  which extends the results for horizontal and vertical lines from S. Dahlke, S. Häuser, G. Steidl, and G. Teschke, *Shearlet coorbit spaces: traces and embeddings in higher dimensions*. *Monatsh. Math.*, 169(1), 15–23, (2013).



## Super-Resolution

Chair: Sinan Gunturk

### 2.50-3.15: The dual approach to non-negative super-resolution: impact on primal reconstruction accuracy

*Bogdan Toader, Stéphane Chrétien & Andrew Thompson*

**Abstract:** We study the problem of super-resolution, where we recover the locations and weights of non-negative point sources from a few samples of their convolution with a Gaussian kernel. It has been recently shown that exact recovery is possible by minimising the total variation norm of the measure. An alternative practical approach is to solve its dual. In this paper, we study the stability of solutions with respect to the solutions to the dual problem. In particular, we establish a relationship between perturbations in the dual variable and the primal variables around the optimiser. This is achieved by applying a quantitative version of the implicit function theorem in a non-trivial way.

### 3.15-3.40: Conditioning of restricted Fourier matrices and super-resolution of MUSIC

*Wenjing Liao & Weilin Li*

**Abstract:** This paper studies stable recovery of a collection of point sources from its noisy  $M + 1$  low-frequency Fourier coefficients. We focus on the super-resolution regime where the minimum separation of the point sources is below  $1/M$ . We propose a separated clumps model where point sources are clustered in far apart sets, and prove an accurate lower bound of the Fourier matrix with nodes restricted to the source locations. This estimate gives rise to a theoretical analysis on the super-resolution limit of the MUSIC algorithm.

### 3.40-4.05: Iterative Discretization of Optimization Problems Related to Superresolution

*Axel Flinth & Pierre Armand Weiss*

**Abstract:** We study an iterative discretization algorithm for solving optimization problems regularized by the total variation norm over the space of Radon measures on a bounded subset of  $\mathbb{R}^d$ . Our main motivation to study this problem is the recovery of sparse atomic measures from linear measurements. Under reasonable regularity conditions, we arrive at a linear convergence rate guarantee.



## Sampling and Fourier analysis

Chair: Arash Amini

### 4.30-4.55: On the Reconstruction of a Class of Signals Bandlimited to a Disc

*Ahmed Zayed*

**Abstract:** Signal reconstruction is one of the most important problems in signal processing and sampling theorems are one of the main tools used for such reconstructions. There is a vast literature on sampling in one and higher dimensions of bandlimited signals. Because the sampling formulas and points depend on the geometry of the domain on which the signals are confined, explicit representations of the reconstruction formulas exist mainly for domains that are geometrically simple, such as intervals or parallelepiped symmetric about the origin. In this talk we derive sampling theorem for the reconstruction of signals that are bandlimited to a disc centered at the origin. This will be done for a more general class of signals than those that are bandlimited in the Fourier transform domain. The sampling points are related to the zeros of the Bessel function.

### 4.55-5.20: The Solvability Complexity Index of Sampling-based Hilbert Transform Approximations

*Volker Pohl & Holger Boche*

**Abstract:** This paper determines the solvability complexity index (SCI) and a corresponding tower of algorithms for the computational problem of calculating the Hilbert transform of a continuous function with finite energy from its samples. It is shown that the SCI of these algorithms is equal to 2 and that the SCI is independent of whether the calculation is done by linear or by general (i.e. linear and/or non-linear) algorithms.



## Sampling and Fourier analysis

Chair: Arash Amini

### 5.20-5.45: The Convolution Word is Tied to the Exponential Kernel Transforms. What is a Parallel Expression for the Other Transforms?

*Abdul Jerri*

**Abstract:** Convolution products are very useful in computations involving exponential kernel transforms, such as the Fourier and Laplace transforms. The feasibility of this method is due to the important property  $e^{\alpha x} e^{\beta x} = e^{(\alpha+\beta)x}$ , that is behind the simple form of the convolution product. Convolution also has the visual property, as it means bending together of the functions. For non-exponential kernel transforms, such as the Hankel transform, the analytical expressions of the Inverse Transform of Two Transforms Product (ITTTP) is quite complicated for use in practical analytical computations. Such difficulty is, mainly, due to the absence of the previously mentioned exponential function property. This is illustrated with a variety of well-known integral transforms. Also, such difficulty is supported by testimonials of experts in the field, such as Ruell Churchill and Ian Sneddon. Therefore, there is a need to return to basics numerical integration, according to the definition of the ITTTP. This is our most recent experience, in cooperation with M. Kamada, in trying to compute the general transform hill functions  $\psi_{R+1}(x)$  associated with the Bessel function kernel, that the speaker had introduced in 1983. They are defined as the  $R$ -times  $J_N$  kernel “convolution parallel” of the gate function.