SampTA 2019

13th International conference on Sampling Theory and Applications

université de BORDEAUX

July 8-12

# Monday July 8, 2.00-5.35 pm

| | | |
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| Wegener | 2.00-3.00 | *Sharpness, Restart and Compressed Sensing Performance*<br>Alexandre d'Aspremont<br>Chair: Pierre Vandergheynst |
| | | **Frame Theory** (invited session)<br>Chair: John Jasper & Dustin Mixon |
| Wegener | 3.00-3.25 | *Equi-isoclinic subspaces from difference sets*<br>Matthew Fickus & Courtney Schmitt |
| Wegener | 3.25-3.50 | *Exact Line Packings from Numerical Solutions*<br>Hans Parshall & Dustin G. Mixon |
| Wegener | 3.50-4.15 | *2- and 3-Covariant Equiangular Tight Frames*<br>Emily King |
| Wegener | 4.15-4.45 | *Coffee Break* |
| Wegener | 4.45-5.10 | *The Zak transform and representations induced from characters of an abelian subgroup*<br>Joseph W. Iverson |
| Wegener | 5.10-5.35 | *A Delsarte-Style Proof of the Bukh-Cox Bound*<br>Mark E. Magsino, Dustin G. Mixon & Hans Parshall |

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| Room | Time | Session |
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| Wegener | 2.00-3.00 | *Sharpness, Restart and Compressed Sensing Performance*<br>Alexandre d'Aspremont<br>Chair: Pierre Vandergheynst |
| | | **Phase retrieval**<br>Chair: Gert Tamberg |
| Edison | 3.00-3.25 | *Conjugate Phase Retrieval in Paley-Wiener Space*<br>Eric S. Weber, Friedrich Littmann & Chun-Kit Lai |
| Edison | 3.25-3.50 | *Phase Retrieval for Wide Band Signals*<br>Philippe Jaming, Karim Kellay & Rolando Perez III |
| Edison | 3.50-4.15 | *Dual-Reference Design for Holographic Phase Retrieval*<br>David A Barmherzig, Ju Sun, Emmanuel Candès, Thomas Joseph Lane & Po-Nan Li |
| | 4.15-45 | *Coffee Break* |
| | | **Probabilistic Methods**<br>Chair: Holger Rauhut |
| Edison | 4.45-5.10 | *Non-Gaussian Random Matrices on Sets: Optimal Tail Dependence and Applications*<br>Halyun Jeong, Xiaowei Li, Yaniv Plan & Ozgur Yilmaz |
| Edison | 5.10-5.35 | *Fast Multitaper Spectral Estimation*<br>Santhosh Karnik, Justin K Romberg & Mark Davenport |
| Edison | 5.55-6.00 | *Monte Carlo wavelets: a randomized approach to frame discretization*<br>Stefano Vigogna, Zeljko Kereta, Valeriya Naumova, Lorenzo Rosasco & Ernesto De Vito |

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# Sharpness, Restart and Compressed Sensing Performance

**Alexandre d'Aspremont** (École Normale Supérieure, France)

**Abstract:** We show that several classical quantities controlling compressed sensing performance directly match parameters controlling algorithmic complexity. We first describe linearly convergent restart schemes on first-order methods using a sharpness assumption. The Lojasievicz inequality shows that sharpness bounds on the minimum of convex optimization problems hold almost generically. Here, we show that sharpness directly controls the performance of restart schemes. For sparse recovery problems, sharpness at the optimum can be written as a condition number, given by the ratio between true signal sparsity and the largest signal size that can be recovered by the observation matrix. Overall, this means that in compressed sensing problems, a single parameter directly controls both computational complexity and recovery performance.

Joint work with Vincent Roulet (University of Washington) and Nicolas Boumal (Princeton University).

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## Frame theory

Chair: John Jasper & Dustin Mixon

### 3:00-3:25: Equi-isoclinic subspaces from difference sets

*Matthew Fickus* & Courtney Schmitt

**Abstract:** Equi-chordal tight fusion frames (ECTFFs) and equi-isoclinic tight fusion frames (EITFFs) are types of optimal packings in Grassmannian spaces. In particular, an ECTFF is an arrangement of equi-dimensional subspaces of a Euclidean space with the property that the smallest chordal distance between any pair of these subspaces is as large as possible. An EITFF is a special type of ECTFF that also happens to be an optimal packing with respect to the spectral distance. In the special case where the subspaces are one-dimensional, both ECTFFs and EITFFs reduce to optimal packings of lines known as equiangular tight frames (ETFs); these lines have minimal coherence, achieving equality in the Welch bound. ETFs are tricky to construct, but several infinite families of them are known. Harmonic ETFs in particular arise by restricting the characters of a finite abelian group to a difference set for that group. Moreover, there is a simple tensor-based method for combining ETFs with orthonormal bases to form EITFFs. It is an open question as to whether every EITFF essentially arises in this way. In this short paper, we preview a new result relating difference sets, harmonic ETFs, ECTFFs and EITFFs. This work expands on other recent results showing that certain harmonic ETFs are comprised of a number of simpler ETFs known as regular simplices; such ETFs arise, for example, from McFarland difference sets as well as the complements of certain Singer difference sets. It is already known that in this situation the subspaces spanned by these regular simplices necessarily form an ECTFF. We recently discovered that these same subspaces form an EITFF in some, but not all, cases. In an upcoming journal article, we shall characterize the properties of the underlying difference sets that lead to EITFFs in this fashion.

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# Frame theory

Chair: John Jasper & Dustin Mixon

### 3:25-3:50: Exact Line Packings from Numerical Solutions

*Hans Parshall* & Dustin G. Mixon

**Abstract:** Recent progress in Zauner's conjecture has leveraged deep conjectures in algebraic number theory to promote numerical line packings to exact and verifiable solutions to the line packing problem. We introduce a numerical-to-exact technique in the real setting that does not require such conjectures. Our approach is completely reproducible, matching Sloane's database of putatively optimal numerical line packings with Mathematica's built-in implementation of cylindrical algebraic decomposition. As a proof of concept, we promote a putatively optimal numerical packing of eight points in the real projective plane to an exact packing, whose optimality we establish in a forthcoming paper.

### 3:50 - 4:15: 2- and 3-Covariant Equiangular Tight Frames

*Emily King*

**Abstract:** Equiangular tight frames (ETFs) are configurations of vectors which are optimally geometrically spread apart and provide resolutions of the identity. Many known constructions of ETFs are group covariant, meaning they result from the action of a group on a vector, like all known constructions of symmetric, informationally complete, positive operator-valued measures. In this short article, some results characterizing the transitivity of the symmetry groups of ETFs will be presented as well as a proof that an infinite class of so-called Gabor-Steiner ETFs are roux lines, where roux lines are a generalization of doubly transitive lines.

### 4:45-5:10: The Zak transform and representations induced from characters of an abelian subgroup

*Joseph W. Iverson*

**Abstract:** We introduce a variant of the Zak transform for a finite group $G$ with respect to a fixed abelian subgroup $H$, and demonstrate a relationship with representations of $G$ induced from characters of $H$. We also show how the Zak transform can be used to study right translations by $H$ in $L^2(G)$, and give some examples of applications for equiangular tight frames.

# Frame theory

Chair: John Jasper & Dustin Mixon

**5:10-5:35: A Delsarte-Style Proof of the Bukh-Cox Bound**

*Mark E. Magsino*, Dustin G. Mixon & Hans Parshall

**Abstract:** The line packing problem is concerned with the optimal packing of points in real or complex projective space so that the minimum distance between points is maximized. Until recently, all bounds on optimal line packings were known to be derivable from Delsarte's linear program. Last year, Bukh and Cox introduced a new bound for the line packing problem using completely different techniques. In this paper, we use ideas from the Bukh-Cox proof to find a new proof of the Welch bound, and then we use ideas from Delsarte's linear program to find a new proof of the Bukh-Cox bound. Hopefully, these unifying principles will lead to further refinements.

# Phase retrieval

Chair: Gert Tamberg

### 3.00-3.25: Conjugate Phase Retrieval in Paley-Wiener Space

*Eric S. Weber*, Friedrich Littmann & Chun-Kit Lai

**Abstract:** We consider the problem of conjugate phase retrieval in Paley-Wiener space $PW_\pi$. The goal of conjugate phase retrieval is to recover a signal $f$ from the magnitudes of linear measurements up to unknown phase factor and unknown conjugate, meaning $f(t)$ and $\overline{f(t)}$ are not necessarily distinguishable from the available data. We show that conjugate phase retrieval can be accomplished in $PW_\pi$ by sampling only on the real line by using structured convolutions. We also show that conjugate phase retrieval can be accomplished in $PW_\pi$ by sampling both $f$ and $f'$ only on the real line. Finally, we show that generically, conjugate phase retrieval can be accomplished by sampling at 3 times the Nyquist rate, whereas phase retrieval requires sampling at 4 times the Nyquist rate.

### 3.25-3.50: Phase Retrieval for Wide Band Signals

Philippe Jaming, Karim Kellay & *Rolando Perez III*

**Abstract:** This study investigates the phase retrieval problem for wideband signals. More precisely, we solve the following problem: given $f \in L^2(\mathbb{R})$ with Fourier transform in $L^2(\mathbb{R}, e^{2c|x|}\mathrm{d}x)$ we determine all functions $g \in L^2(\mathbb{R})$ with Fourier transform in $L^2(\mathbb{R}, e^{2c|x|}\mathrm{d}x)$, such that $|f(x)| = |g(x)|$ for all $x \in \mathbb{R}$. To do so, we translate the problem into a phase retrieval problem for functions in Hardy spaces on the disc and use the ineer-outer factorization.

### 3.50-4.15: Dual-Reference Design for Holographic Phase Retrieval

*David A. Barmherzig*, Ju Sun, Emmanuel Candès, Thomas Joseph Lane & Po-Nan Li

**Abstract:** A new reference design is introduced for Holographic Coherent Diffraction Imaging. This consists of two reference portions - being "block" and "pinhole" shaped regions - adjacent to the imaging specimen. Expected error analysis on data following a Poisson shot noise model shows that the dual-reference scheme produces smaller weighting of error across the frequency spectrum than does the leading single-reference schemes. Numerical experiments on simulated data also shows the dual-reference scheme achieving a smaller recovery error than the leading single-reference schemes.

# Probabilistic Methods

Chair: Holger Rauhut

### 4.45-5.10: Non-Gaussian Random Matrices on Sets: Optimal Tail Dependence and Applications

*Halyun Jeong*, Xiaowei Li, Yaniv Plan & Ozgur Yilmaz

**Abstract:** Random linear mappings play a large role in modern signal processing and machine learning. For example, multiplication by a Gaussian matrix can preserve the geometry of a set while reducing the dimension. Non-gaussian random mappings are attractive in practice for several reasons, including improved computational cost. On the other hand, mappings of interest often have heavier tails than Gaussian, which can lead to worse performance, i.e., less accurate preservation of the geometry of the set. In the sub-Gaussian case, the size of the tail is measured with the sub-gaussian parameter, but the dependency has not been fully understood yet. We present the optimal dependency on the sub-gaussian parameter and prove it through a new version of Bernstein's inequality. We also illustrate popular applications whose theoretical guarantees can be improved by our results.

### 5.10-5.35: Fast Multitaper Spectral Estimation

*Santhosh Karnik*, Justin K Romberg & Mark Davenport

**Abstract:** Thomson's multitaper method using discrete prolate spheroidal sequences (DPSSs) is a widely used technique for spectral estimation. For a signal of length $N$, Thomson's method requires selecting a bandwidth parameter $W$, and then uses $K \sim 2NW$ tapers. The computational cost of evaluating the multitaper estimate at $N$ grid frequencies is $O\big(KN\log(N)\big)$. It has been shown that the choice of $W$ and $K$ which minimizes the MSE of the multitaper estimate is $W = O(N^{-1/5})$ and $K = O(N^{4/5})$. This choice would require a computational cost of $O\big(N^{9/5}\log(N)\big)$. We demonstrate an $\varepsilon$-approximation to the multitaper estimate which can be evaluated at $N$ grid frequencies using $O\big(N\log^2(N)\log(1/\varepsilon)\big)$ operations.

### 5.35-6.00: Monte Carlo wavelets: a randomized approach to frame discretization

Stefano Vigogna, *Zeljko Kereta*, Valeriya Naumova, Lorenzo Rosasco & Ernesto De Vito

**Abstract:** In this paper we propose and study a family of continuous wavelets on general domains, and a corresponding stochastic discretization that we call Monte Carlo wavelets. First, using tools from the theory of reproducing kernel Hilbert spaces and associated integral operators, we define continuous wavelets using spectral calculus. Then, we propose a stochastic discretization based on Monte Carlo estimates of the integral operators. Using concentration of measure results, we establish the convergence of such a discretization and derive convergence rates under natural regularity assumptions.